If the centre of inversion is a node, the inverse is an $(m-2)$-circular $(2m-2)$-ic, having two real, coincident or imaginary points at infinity, according as the node is a crunode, cusp or acnode. In general, if the centre of inversion is a multiple point of the s order, the inverse is an $(m-s)$-circular $(2m-s)$-ic, with s real or imaginary points at infinity corresponding to the s branches of the given curve at the centre of inversion.

14. It is evident geometrically that a node or cusp, *not* at the centre of inversion, inverts into a similar point, so that the possession of these singularities affords a sub-classification within the circular orders such that a curve and its inverse belong to the same sub-class.

Thus a non-singular bi-circular quartic can invert only into non-singular bi-circular quartics or circular cubics. The crunodal bi-circular quartics and circular cubics, with the hyperbolas, form another sub-class the members of which invert only one into another. The cuspidal bi-circular quartics and circular cubics, together with the parabola, form another class of curves bearing the same mutual relations, and finally the acnodal varieties of these higher curves are similarly associated with the ellipse. We have an example in the case of the cissoid of equation (7), which has a cusp at the point $(-a, 0)$; this cusp inverts into a cusp, which, since the modulus employed was a^2, is situated at the same point. Accordingly, if we transfer the origin in eq. (8) to the point $(-a, 0)$, we have the equation

$$r^4 - ar^2x - 2a^2y^2 = 0,$$

showing that the curve has a cusp at which the tangent is the axis of x. If we invert this curve with respect to its present origin, the cusp, we shall have the parabola

$$a^2 - ax - 2y^2 = 0.$$

DETERMINATION OF THE LOCUS OF O. (SEE FIG. ON P.22.)

BY CHRISTINE LADD, UNION SPRINGS, N. Y.

THE perpendicular on γ from $\alpha'\beta'\gamma'$ is

$$\alpha\beta' - \beta\alpha' = (\beta\gamma' - \gamma\beta')\cos B + (\gamma\alpha' - \alpha\gamma')\cos A,$$

and the line joining its intersection with γ to the vertex C is

$$\alpha(\beta' + \gamma'\cos A) - \beta(\alpha' + \gamma'\cos B) = 0.$$

The condition that the three lines of this kind meet in a point is

$$(\alpha' + \beta'\cos C)(\beta' + \gamma'\cos A)(\gamma' + \alpha'\cos B)$$
$$= (\alpha'\cos C + \beta')(\beta'\cos A + \gamma')(\gamma'\cos B + \alpha'),$$

or the locus of $\alpha'\beta'\gamma'$ is

$$\alpha(\beta^2-\gamma^2)(\cos A-\cos B\cos C)+\beta(\gamma^2-\alpha^2)(\cos B-\cos C\cos A)$$
$$+\gamma(\alpha^2-\beta^2)(\cos C-\cos A\cos B)=0. \ldots\ldots (1)$$

This equation represents a cubic which passes through the vertices of the triangle and cuts the sides α, β, γ again at their respective intersections with

$$\beta(\cos C+\cos A\cos B) = \gamma(\cos B+\cos C\cos A),$$
$$\gamma(\cos A+\cos B\cos C) = \alpha(\cos C+\cos A\cos B),$$
$$\alpha(\cos B+\cos C\cos A) = \beta(\cos A+\cos B\cos C).$$

The equation (1) is satisfied by the coordinates 1, 1, 1; $\cos A$, $\cos B$, $\cos C$; and $\cos B\cos C$, $\cos C\cos A$, $\cos A\cos B$; hence the curve passes through the centres of the inscribed and circumscribed circles and through the orthogonal centre of the triangle.

If $A = B$, (1) becomes

$$(\alpha-\beta)[c(\gamma^2+\alpha\beta)-a(\alpha\gamma+\beta\gamma)] = 0.$$

$\alpha-\beta$ is the bisector of the angle C and, on applying the criterion (Todhunter's Conic Sections, p. 306),

$$c(\gamma^2+\alpha\beta)-a(\alpha\beta+\beta\gamma)=0$$

is found to be an hyperbola. One branch passes through A and the middle point of AC, the other, through B and the middle point of BC. Its transverse axis is parallel to AB. If $A = B = C$, (1) becomes

$$(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)=0,$$

the bisectors of the angles of the triangle.

Note by Artemas Martin. — There is an omission in my solution of problem 139, p. 29, Vol. IV. In the first line the coefficients $1+c^2$ and $1-c^2$ should be divided by c^2. In the third line write $(2c^2-1)\div c^2$ for $(2-c^2)$ and $(1-c^2)\div c^2$ for $(1-c^2)$.

The corrected result is

$$\Delta = \frac{4\pi a^3}{3ceS}\left[ce(1-e^2)^{\frac{1}{2}} + \left(\frac{2c^2-1}{c^2}\right)[E'(c)] + \left(\frac{1-c^2}{c^2}\right)[F'(c)]\right].$$

Query, by Prof. A. Hall. — The approximate value of the definite integral

$$\int_0^{\frac{\pi}{2}}\sqrt{\sin\varphi}\,.\,d\varphi$$

is 1.198 ....: Is there a convenient way of computing this numerical value?